Cold Regions Science and Technology 54 (2008) 183–192

Contents lists available at ScienceDirect

Cold Regions Science and Technology

journal homepage: www.elsevier.com/locate/coldregions

Statistical runout modeling of snow avalanches using GIS in Glacier National Park, Canada

D. Delparte [a,*], B. Jamieson [b,1], N. Waters [c,2]

[a] Department of Geography, University of Calgary, 2500 University Dr NW, Calgary, AB, Canada T2N 1N4
[b] Department of Civil Engineering and the Department of Geoscience, 2500 University Dr NW, University of Calgary, Calgary, AB, Canada T2N 1N4
[c] Department of Geography, George Mason University, 4400 University Drive MS 1E2, Fairfax, Virginia, 22030, United States

ARTICLE INFO

Article history:
Received 20 September 2007
Accepted 28 July 2008

Keywords:
Snow avalanches
Hazard mapping
Geographic Information Systems
Digital terrain model

ABSTRACT

Using models to estimate snow avalanche runout distance is useful for areas where there is a lack of historical avalanche observations and no obvious physical signs of avalanche activity. Along roadways, details of avalanche runout are often recorded; however, in Canada, backcountry areas typically used by recreationists may not have a recorded history of avalanche activity or runout distances. Knowledge of predicted runout extents mapped in Geographic Information Systems (GIS) has the potential to inform backcountry users on route selection and decision making pertaining to slopes for skiing, snowboarding, ice climbing and snowmobiling. The Rogers Pass area in Glacier National Park, British Columbia, Canada provides an ideal location for studying well documented avalanche paths that impact the Trans Canada Highway, as well as representing a backcountry area that is a popular ski touring destination in Canada. A statistical approach using the alpha–beta runout model, first developed in Norway, has been adapted for use in Rogers Pass. Topographic parameters from well-known avalanche paths along the Trans Canada Highway corridor, with a historical record of over 40 years, have been extracted with GIS and used to calibrate the alpha–beta runout model. This model is then applied to an avalanche path in the Glacier National Park backcountry. A high resolution Digital Elevation Model (DEM) was created for the study area using digital stereo photogrammetry. A comparison of model calculations using the higher resolution dataset and a lower resolution dataset did not reveal any significant difference between the model parameters.

1. Introduction

Determining snow avalanche runout extent is an important consideration for mapping avalanche hazard for backcountry users, transportation corridors and other human infrastructure. The runout zone is the portion of the avalanche path where large avalanches begin to slow down and deposition of snow and entrained material occurs. The threshold for mapping runout in backcountry ski operations involves identifying a runout extent for a return period of less than 10 years (Canadian Avalanche Association, 2002). Runout zones can be identified through a combination of field observations, historical records, meteorological data and analysis of aerial photos and topographic maps for vegetative and geomorphic evidence (Canadian Avalanche Association, 2002; Mears, 1992; Weir, 2002). In areas where historical observations and field evidence are lacking, estimating avalanche extent may be difficult and runout models provide an option for estimating runout. Snow avalanche runout modeling is generally accomplished with statistical models, physically based models or a combination of the two approaches. The statistical models provide an estimate for runout along the centreline of the avalanche path and are limited in that they do not indicate the lateral extent of avalanches.

In 2004 Parks Canada developed an Avalanche Terrain Exposure Scale (ATES) based upon terrain and land cover characteristics to identify avalanche susceptible areas in the backcountry (Statham et al., 2006). Using a defining set of terrain based criteria, avalanche professionals designate “simple”, “challenging” and “complex” avalanche terrain. Elements of determining the exposure scale rely upon the identification of runout. ATES is also used as part of the Avaluator trip planner that takes into account the current avalanche bulletin (Haegeli and McCammon, 2006). Varying levels of caution are recommended depending upon the level of avalanche danger and the type of terrain the user is travelling in. Runout models offer the potential for ATES-based runout mapping to aid backcountry users in reducing risk.

Dynamic models are adept at indicating velocity and impact pressures along with avalanche runout and are especially suited for analysis where defence structures would be situated or impacts to

* Corresponding author. Tel.: +1 403 220 5584; fax: +1 403 2826561.
E-mail addresses: ddelparte@selkirk.ca (D. Delparte), bruce.jamieson@ucalgary.ca (B. Jamieson), nwaters@gmu.edu (N. Waters).
[1] Tel.: +1 403 220 7479; fax: +1 403 282 7026.
[2] Tel.: +1 703 993 4687; fax: +1 703 993 1216.

0165-232X/$ – see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.coldregions.2008.07.006

forest resources or infrastructure are concerned; however, they require estimates of friction coefficients and release mass which may be unavailable or difficult to estimate in remote areas or areas with varying terrain cover and unknown snow depth. Small variations in these parameters can lead to great discrepancies in estimating runout distance (Lied, 1998). Statistical models using regression equations with simple topographic inputs, first introduced by Bovis and Mears (1976), are able to predict maximum runout but do not provide estimates of avalanche size, speed, force or lateral extent. For the purposes of hazard mapping for recreational users, the runout predicted by statistical models is sufficient to identify the extent of avalanche paths to a backcountry user. The alpha–beta regression runout model (Bakkehoi et al., 1983; Lied and Bakkehoi, 1980; Lied et al., 1989), initially applied in Norway by the Norwegian Geotechnical Institute (NGI) and applied in other parts of the world (Fujisawa et al., 1993; Furdada and Vilaplana, 1998; Johannesson, 1998; Jones and Jamieson, 2004; Lied et al., 1995) is well suited to topographic mapping. A modified version of the Norwegian model, the runout-ratio model, has been developed for extreme value distributions. The main assumption is that extreme avalanche events follow a Gumbel distribution as opposed to an assumed normal distribution of the residuals from a regression model (McClung, 2000; McClung and Lied, 1987; McClung and Mears, 1991; McClung and Mears, 1995; McClung et al., 1989). Comparisons between the runout-ratio model and the regression model have been made using different datasets. In Iceland, runout-ratio models did not show an improvement over regression models (Johannesson, 1998). For short path avalanche data in Canada, the runout-ratio model often predicted longer runout distances than the alpha–beta model, especially for large non-exceedence probabilities (Jones and Jamieson, 2004). For this study, the alpha–beta approach is used for predicting avalanche runout.

The terrain inputs required for the regression runout model are typically taken from topographic maps and field survey measurements; however, in Europe, Geographic Information Systems and digital elevation model (DEM) data (based on data with contour intervals of 20 m) have been used to extract these parameters (Furdada and Vilaplana, 1998; Lied et al., 1989; Lied and Toppe, 1989; Toppe, 1987), with the assumption that better or higher resolution DEM data would improve the models. One part of this study examines the assumption that higher resolution DEM data improves regression runout models by running a comparison between the results obtained in an alpha–beta regression analysis between low resolution DEM data and high resolution DEM data.

Variations of the alpha–beta regression model equation have been produced for diverse mountain ranges. This study focuses on developing a regression model specifically for the Columbia Mountains, Glacier National Park, British Columbia, Canada. Utilizing an extensive historical database of observed maximum runout records of over 40 years supplemented with expert knowledge along the Trans Canada Highway (TCH) corridor. The model is applied to an avalanche path in the Connaught Creek drainage, a popular backcountry ski area, as an example of the model's applicability for predicting runout extent in lesser known areas in the park.

2. Study area and data sources

2.1. Study area

The location of the study area is in Glacier National Park (GNP), British Columbia, Canada, which is situated in the Columbia Mountains of western Canada. GNP is located along the Trans Canada Highway and the main line of the Canadian Pacific Railway, approximately 350 km west of Calgary, Alberta. This area is locally known as Rogers Pass and is located between the towns of Golden and Revelstoke, British Columbia. The highway provides easy access to backcountry skiing in the park. The terrain of GNP is characterised by steep-sided glaciated valleys, rugged high peaks and expansive glaciers. Elevations in the park range from 800 m in the valley bottom to approximately 3400 m for the highest mountain peaks. Along the highway corridor, covering a distance of 45 km, there are 144 avalanche paths, which are monitored by the Parks Canada avalanche control program. Detailed maps of the study area are available in Delparte (2008) and of the highway corridor avalanche paths in Schleiss (1989). Parks Canada operates one of the world's largest mobile avalanche control programs to mitigate avalanche hazard along the Trans Canada Highway and portions of the CPR that are unprotected by tunnels and snow sheds. The exposure of the highway and its importance as a transportation corridor led to the formation of an avalanche control program before the construction of the highway began in 1959 (Woods, 1983).

To counteract the impact of avalanche runout on the highway and exposed sections of the rail line, additional defensive structures including snow sheds, diversion dams, mounds and other barriers have been constructed. Despite these measures, there are avalanche paths that reach the highway in the valley bottom and in some cases will cross the valley floor and run up the opposite slope. To control avalanche paths that impact the transportation corridor, weather, snowpack and avalanche observations are evaluated by avalanche forecasters and, if necessary, the TCH and CPR are closed while avalanches are triggered artificially with assistance from the Canadian artillery. Road crews then clear snow and debris from the highway. Closures total about 100 h per winter and have significant economic impacts.

2.2. Data sources

For the Rogers Pass highway corridor, a consistent database of the maximum observed extent of natural and triggered avalanches has been collected for over 40 years. For this study the database was supplemented with first hand expert knowledge from the senior avalanche officer, Bruce McMahon, who has over 20 years of operational experience at Rogers Pass. His expert knowledge and experience with the runout database were essential as he was able to recall observations made or refer to original field notes on recorded maximum runout extents. In addition, his detailed knowledge of terrain characteristics and the avalanche activity in the avalanche paths along the highway, which is not available in any database, was a critical component of the mapping work undertaken, particularly for verifying mapped data in the GIS.

This study is based on extracting topographic parameters from expert-defined avalanche paths, starting zones and path centrelines (profiles) to estimate runout from both a high and low resolution digital elevation model. To map accurately the expert-defined avalanche path outlines, starting zones and centrelines of avalanche paths with maximum runout indicated by the farthest reach of the line, digital stereo vector mapping with a photogrammetry workstation using SOCET SET® accommodated the digitising of the data. This process and a map of the areas digitized is described in greater detail in Delparte (2008).

The high resolution DEM was created using a process of digital photogrammetry whereby air photos that were taken in the fall of 2004, at a scale of 1:30,000, were scanned, geo-corrected and manually processed to develop a higher resolution DEM (Delparte, 2008). An interpolated grid with a 5 m horizontal resolution was created using GIS. Due to the time consuming nature of building a DEM with this method, only a portion of the terrain covered by the avalanche paths along the highway corridor was processed, along with two popular backcountry touring areas the Connaught Creek and Asulkan Drainages. The low resolution DEM with an interpolated horizontal resolution of 25 m was obtained from the Province of British Columbia, Base Mapping and Geomatic Services. The low resolution DEM covers the entire study area.

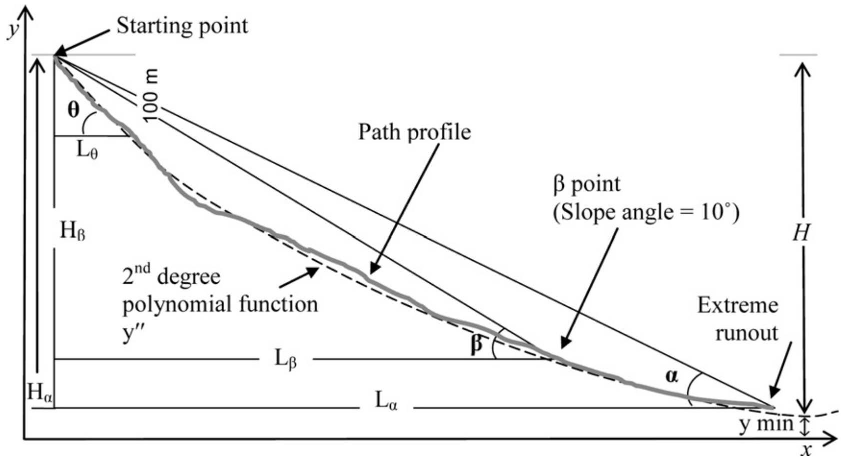

Fig. 1. Alpha–beta runout model. The solid line represents the centreline of the avalanche path profile from the DEM. The dashed line is the 2nd degree polynomial derived from the avalanche profile coordinate points. θ is the average inclination of the starting zone in the top 100 vertical meters (arctan($100/L_\theta$))of the avalanche path. H represents vertical drop as measured on the 2nd degree polynomial from the polynomial's intersection with the y axis at the top of the avalanche path to the minimum point measure on the curve of the polynomial function. β is the average angle (arctan(H_β/L_β)) from the top of the avalanche path to a point at which the slope angle reaches 10° (the β point) on the DEM profile. α measures the average angle (arctan(H_α/L_α))from the top of the avalanche path to the observed maximum runout position.

2.3. Description of the avalanche path dataset

For the analysis a dataset of 35 avalanche paths was used. Since the alpha–beta regression model is not intended to include run-up, only avalanches with a maximum observed runout that involved a run-up the opposing slope of less than 25 vertical metres, from the valley bottom up to the recorded stopping point, were included in the analysis. Of those avalanche paths that do run up the opposing slope less than 25 vertical metres, this difference is lessened as the winter progresses and the deposition of snow fills the depression and reduces the run-up amount. All the paths that included some run-up had a measured run-up that accounted for less than 2.5% of their total vertical drop. The median run-up of the avalanche paths used for the analysis was 1.1 m, which is less than the DEM vertical accuracy of +/−3 m.

The following topographic parameters were extracted based on the success of previously cited alpha–beta studies in estimating maximum runout. These parameters (H, β and $Hy''\theta$) have proven useful in regression models for estimating runout maximums (α) in mountainous regions around the world, as cited in the Introduction, and are illustrated in Fig. 1.

α (alpha): is the average gradient of avalanche path from the top of the avalanche path to the end of the maximum observed runout position where α=arctan (H_α/L_α). Coordinates were extracted from the highest and lowest points from the actual DEM (high and low resolution) on DEM avalanche profile line to yield vertical drop (H_α) and the total horizontal distance (L_α). α (observed) is calculated based upon where the expert positioned the maximum observed runout position in the GIS.

β (beta): is the average gradient (where β=arctan (H_β/L_β)) of the avalanche profile from the top of the path to the point on the avalanche profile within the runout zone where slope reaches 10° (beta point). All slopes of 10° and less were mapped in the GIS and the intersection of the DEM avalanche profile and the down slope point at which it crossed the 10° slope threshold in the runout zone was recorded as the beta point. The beta point was initially thought to represent the point at which large avalanches begin to retard and deposition occurs (Bakkehoi et al., 1983; Lied and Bakkehoi, 1980); however, the β angle provides the best characterization of the inclination of the avalanche track and provides a simple reference for performing runout calculations (Bakkehoi et al., 1983; Harbitz et al., 2001; Lied et al., 1995). Coordinates were extracted from the high and low DEM resolution points at the top of the avalanche path and the beta point to calculate the beta angles. A similar procedure was used to identify a β_{24} point where the slope decreased to 24°, since Jones and Jamieson (2004) found that β_{24} yielded more accurate runout estimates for a Canadian dataset of short slopes.

θ (theta): corresponds to the inclination of starting zone in the top 100 m of the release zone where θ=arctan($100/L_\theta$). On the high and low resolution DEM, a point was created using GIS within the top vertical 100 m (+/−5 m) of the avalanche profile. The coordinates were extracted at this 100 m elevation point along with the coordinates from the top of the path line to yield exact vertical height and horizontal length to provide a measure of average inclination of the starting zone.

y'' (curvature): represents the topographic profile of the avalanche path from the 2nd degree polynomial function. All horizontal distance and elevation data points were collected from along the avalanche profile from the top of the expert identified avalanche path to the expert identified maximum recorded runout point. The file was imported into a statistical package (SPSS version 15.0) and the 2nd degree polynomial equation line of best fit was determined. The coefficient of x^2 was taken from the equation and multiplied by 2 in order to produce the second derivative constant (y'') that provided a value for the curvature. The procedure was performed for both the high and low resolution datasets.

H (vertical displacement): is the total vertical displacement as measured on the 2nd degree polynomial function. Vertical displacement is measured as the difference between the top of the avalanche

Table 1
Descriptive statistics of the topographic parameters used in the regression analysis from both high and low resolution avalanche path datasets

Topographic parameters	N	Mean		Standard deviation		Range of values	
		Low res.	High res.	Low res.	High res.	Low res.	High res.
α (°)	35	28.4	28.5	3.2	3.3	21.4–34.3	21.6–34.5
β (°)	35	30.4	30.6	3.2	3.3	22.7–35.3	22.9–35.4
θ (°)	35	40.1	41.0	5.7	6.0	29.9–56.0	31.6–58.1
H (m)	35	1165	1187	357	381	400–1761	398–2040
y'' (m^{-1})	35	3.6×10^{-5}	3.6×10^{-5}	2.4×10^{-5}	2.5×10^{-5}	6.0×10^{-5} 1.2×10^{-3}	5.0×10^{-5} 1.3×10^{-3}
$Hy''\theta$ (°)	35	13.9	14.4	4.8	5.1	4.4–24.3	4.2–24.6
Vertical drop (m)	35	939	946	261	261	360–1365	356–1376
Surface length (m)	35	2036	2044	615	615	740–3108	740–3118

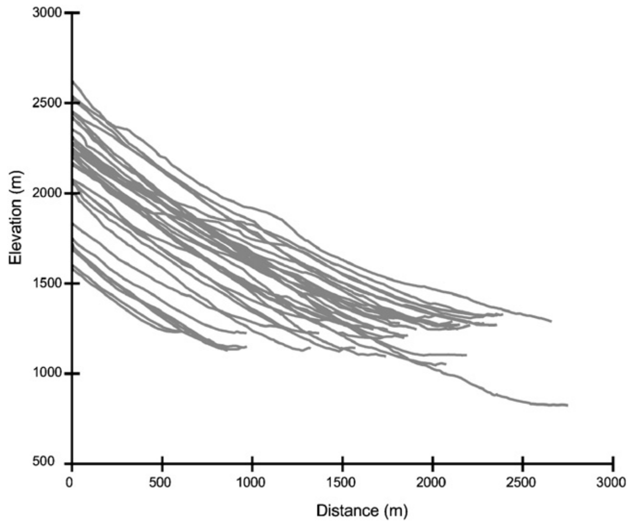

Fig. 2. Snow avalanche profiles of the 35 paths used to develop the alpha–beta runout model for Glacier National Park. Each line indicates the elevation of the expert identified avalanche path from starting point to the observed maximum runout distance in the valley bottom.

path at the y intercept to the minimum point on the 2nd degree polynomial function, where $y'=0$.

Table 1 summarizes the mean, standard deviation and range of the variables required for the regression analysis for both high and low resolution datasets. In addition, the measured vertical drop and profile surface lengths are included to aid in describing the dataset. Note that the alpha and beta angles for the low resolution dataset slightly underestimate slope angle compared to the corresponding angles for the high resolution dataset. The vertical drop from the high resolution dataset ranges from 356 to 1376 m. The measure of H, which indicates vertical drop based on the best fitted parabola, has a higher value than the measured vertical drop, indicating an elevation difference in the runout zone between the 2nd degree polynomial and the DEM profile which is usual (Bakkehoi et al., 1983). H values also display greater discrepancy between high and low resolution datasets. Surface lengths for the high resolution dataset range from 740 m to 3118 m. There is little difference between the mean high and low resolution values for the y'' topographic parameter. Differences in the $Hy''\theta$ topographic parameter are most likely a result of the difference in H between the high and low resolution datasets. Overall the differences between the values of the topographic parameters for the high and low resolution DEM are minimal.

3. Methods

Avalanche centrelines and maximum observed runout positions were digitized based upon expert knowledge from the top of the starting zone to the observed maximum runout position. Digitized linework was referenced to both the high and low resolution DEM data (Delparte, 2008, pp. 88–89). ArcGIS® v 9.2 was used to extract relevant coordinate points into a spreadsheet to perform the calculations required for the regression model.

3.1. Avalanche profiles and determining the model of best fit

Determining the model of best fit for each of the avalanche profiles was essential for determining the curvature and vertical displacement variables for the alpha–beta runout model. Fig. 2 highlights the DEM avalanche profiles used for the regression model calculations. Many of the profiles exhibit a “hockey-stick” shape as discussed in Jones and Jamieson (2004) where there is an abrupt slope change in the runout zone as the slope reaches the alluvium in the valley bottom. Each topographic avalanche profile was fitted with 2nd and 4th degree polynomial functions to derive a profile of best fit. The 2nd degree function resulted in R^2 values of at least 0.993 for the profiles; however, when the 2nd degree function was plotted on a graph it was apparent that the 2nd degree function did not accurately represent the lower portion of the DEM avalanche profile. For example, in Fig. 3, the avalanche profile for path A is represented with a solid black line. The 2nd and 4th degree polynomial functions are indicated respectively in dashed and dotted lines. It was apparent that the second degree function, although it had a high R^2 value, did not represent the bottom of the avalanche profile as well as the 4th degree function. The same issue was observed for the low resolution set of avalanche profiles. These findings correspond to those in Furdada and Vilaplana (1998), who found a similar problem fitting a 2nd degree function in the lower portion of the runout for some of their DEM profile dataset in the western Catalan Pyrenees in Spain. Despite the high R^2 value for the 2nd degree function, the 4th degree polynomial function provided a better fit in the lower portion of the avalanche path than the 2nd

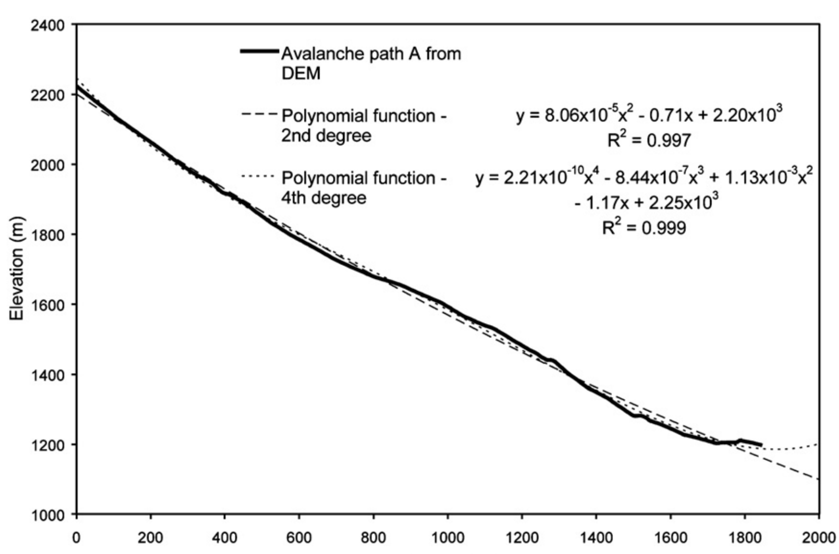

Fig. 3. Avalanche profile for path A with best fitted 2nd and 4th degree polynomials. 4th degree polynomial reveals a better fit than the 2nd degree in the runout zone.

Table 2
Comparison of regression coefficients for high and low resolution DEMs (Eqs. (1) and (2))

Parameter	High resolution		Low resolution		*t*-value	*p* value
	Coeff.	Std. err.	Coeff.	Std. err.		
β	0.800	0.036	0.826	0.034	0.525	0.601
H	1.4×10^{-3}	4.4×10^{-4}	1.3×10^{-3}	4.4×10^{-4}	0.245	0.807
$Hy''\theta$	0.165	0.049	0.131	0.049	0.491	0.625

degree polynomial function for many of the avalanche paths. Due to this finding, we decided to test H and $Hy''\theta$ variables calculated on the 2nd degree polynomial as well as H and $Hy''\theta$ from the better fitting 4th degree polynomial in the regression model. Originally, Lied and Bakkehoi (1980) used a 4th degree function to describe the terrain profile of the avalanche path but dropped it in favour of using a 2nd degree function as it appeared to fit their data just as well as the 4th degree function. The difference in fit between the 2nd and 4th degree polynomials is further analyzed in Appendix A.

4. Results and analyses

To develop an alpha–beta regression model specific to Glacier National Park from the well-known avalanche paths along the highway corridor, we initially tested correlations between the variables in Table 1 for both high and low resolution datasets. Subsequently, we tested variations of the regression model and performed significance tests to determine if there was any difference between the equations derived from the high and low resolution datasets. This included a modification of the β point variable where the definition was changed to indicate a point on the avalanche profile where the slope reached 24° (instead of 10°) in the runout zone. An initial analysis of the dataset through the regression model revealed that the constant value was not significant. This corresponds to findings from some other studies where the constant value for the alpha–beta regression model has been identified as not significant and dropped (Johannesson, 1998; McClung and Mears, 1991; McClung et al., 1989). Thus the model was run again, forcing the intercept through the origin and these results are presented below. The final step was to validate the model results using a statistical procedure known as leave-one-out (LOO) cross-validation.

4.1. Alpha–beta runout model results

Determining an alpha–beta regression model using β values from the DEM profile and variables H and $Hy''\theta$ based on 4th order polynomials resulted in H and $Hy''\theta$ being determined insignificant and beta was isolated as the only significant predicator of alpha. Regression variables of H and $Hy''\theta$ based on 2nd order polynomials did yield significant results as predictors in the regression model and are presented below. The regression model was based on the 35 avalanche profiles for both high and low resolution datasets. The variables of β, H and $Hy''\theta$ were used to determine a regression equation for $\hat{\alpha}$ (predicted). In addition, a simplified formula based on only the β parameter was developed as has been common practice in other studies.

The results for the three predictor model forced through the origin for high and low resolution datasets are as follows (where R^2 is the coefficient of determination and s is the standard deviation of the error):

High resolution:

$$\hat{\alpha}=0.800\beta+1.42\times10^{-3}H+0.165Hy''\theta \quad (1)$$

$$R^2=0.923, s=0.941^{\circ}, n=35$$

where the p values for each parameter are:

$$\beta: p=10^{-21}, H: p=2\times10^{-3}, Hy''\theta: p=2\times10^{-3}$$

Low resolution:

$$\hat{\alpha}=0.826\beta+1.27\times10^{-3}H+0.131Hy''\theta \quad (2)$$

$$R^2=0.919, s=0.939^{\circ}, n=35$$

where the p values for each parameter are:

$$\beta: p=10^{-22}, H: p=7\times10^{-3}, Hy''\theta: p=1\times10^{-2}$$

The 3 predictor alpha–beta runout model has been simplified in other countries based on the strength of the relationship between alpha and beta. A simplified regression equation offers an ease of use in developing GIS applications to automate mapping of runout in the backcountry. The equations follow for the two datasets:

High resolution:

$$\hat{\alpha}=0.933\beta \quad (3)$$

$$R^2=0.889, s=1.098^{\circ}, n=35$$

where the significance value for β is: $p=10^{-50}$

Low resolution:

$$\hat{\alpha}=0.934\beta \quad (4)$$

$$R^2=0.891, s=1.051^{\circ}, n=35$$

where the significance value for β is: $p=10^{-50}$

To test if there was a significant difference between the regression coefficients for the high and low resolution regression models for the study area, Eq. (5) was used (Paternoster et al., 1998). As indicated by the test of the coefficients in Tables 2 and 3, there was no significant difference between the coefficients and hence there is no significant

Table 3
Comparison of regression coefficients for high and low resolution DEMs (Eqs. (3) and (4))

Parameter	High resolution		Low resolution		*t*-value	*p* value
	Coeff.	Std. err.	Coeff.	Std. err.		
β	0.933	6.0×10^{-3}	0.934	5.8×10^{-3}	0.119	0.906

Table 4
Analysis of MSE values for the predicted alpha from the regression model equations and average horizontal and elevation distance from the observed alpha as measured on the DEM profile

Equation	MSE	Average distance from observed (m)	Average elevation distance from observed (m)
3 predictor Eq. (1)	0.8°	51	8
Beta only Eq. (3)	1.2°	58	8
β_{24} Eq. (6)	1.4°	79	9

Table 5
Leave-one-out cross-validation error estimations for the alpha–beta runout model equations

	3 predictor (Eq. (1))	Beta only (Eq. (3))	β_{24} (Eq. (6))
MSE	1.2°	1.3°	1.6°
Mean absolute LOO-error	0.9°	1.0°	1.0°
RMS LOO-error	1.1°	1.1°	1.3°

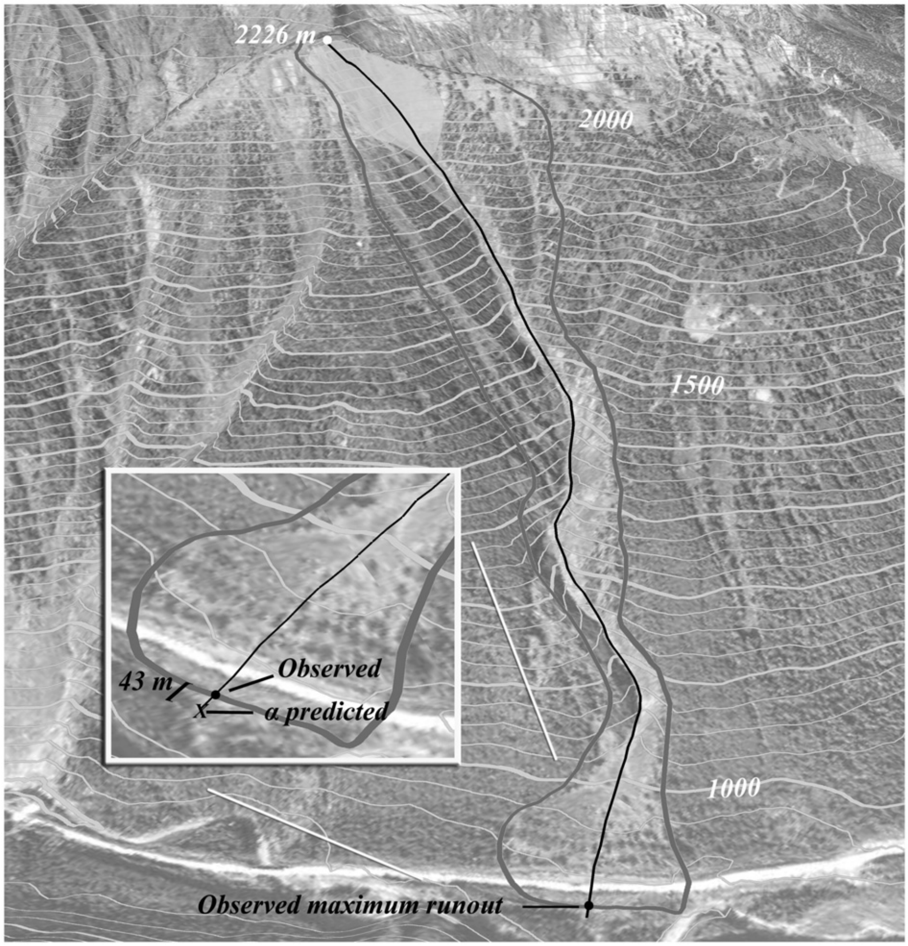

Fig. 4. Avalanche Path B along the Trans Canada highway corridor. Dark centreline highlights the path profile. Starting zone is identified at the top of the path with light grey shading. The bottom portion of the profile centreline extends slightly past the expert identified maximum runout showing the predicted runout from the alpha–beta regression model (inset). Image is draped on the low resolution DEM.

difference between the regression equations derived for the high and low resolution datasets where the intercept is forced through the origin.

$$t=\frac{b_1-b_2}{\sqrt{\mathrm{SE}b_1^2+\mathrm{SE}\,b_2^2{}_{363}}} \tag{5}$$

where b_1 and b_2 represent the coefficients for the respective high and low resolution regression equations and SEb_1 and SEb_2 represent the standard error measures for the coefficients being tested.

The equation for the β_{24} only regression model with β at 24° and an intercept forced through the origin follows for the two datasets:

High resolution:

$$\hat{\alpha}=0.878\beta_{24} \tag{6}$$

R^2=0.865, s=1.211°, n=35

where the p value for β_{24} is: $p=10^{-48}$

Low resolution:

$$\hat{\alpha}=0.881\beta_{24} \tag{7}$$

R^2=0.870, s=1.152°, n=35

where the p value for β_{24} is: $p=10^{-49}$

The regressions using β_{10} show a higher R^2 and lower standard error of estimation than the regressions using β_{24}, which are not analyzed further.

Table 4 presents an analysis of the MSE (mean square error) values comparing the average squared difference between observed alpha values to the predicted values for the 35 avalanche paths in the high resolution dataset. The 3 predictor regression Eq. (1) provides a slightly better fit than the beta only regression Eq. (3). The least amount of measured difference is found in elevation from the different regression equations.

The average difference between the beta only regression (Eq. (3)) and 3 predictor model (Eq. (1)) alpha values are less than 0.5°. With an average path height of 950 m and average alpha of 28.5, a 0.5° difference results in a distance of just under 40 m.

4.2. Validation

In order to validate the alpha–beta regression models, a procedure of leave-one-out (LOO) cross-validation, was used to test the high resolution dataset regression equations with β, H and $Hy''\theta$ as predictors and the β-only regression models. The “Leave-One-Out”

Table 6

Predicted alpha and maximum observed runout ground positions (horizontal distance and elevation) for avalanche path B

Equation	$\hat{\alpha}$ Predicted (°)	Distance (m)	Elevation (m)
3 predictor Eq. (2)	23.4	2979	936
Beta only Eq. (4)	23.2	3049	922
β_{24} Eq. (7)	22.7	3141	914
Alpha observed	23.7	2936	939

(LOO) cross-validation method involves calculating the regression model without one of the observations and repeating for each subsequent observation (Wilks, 1995, pp. 194–198).

Software from Schenley Park Research Inc., Vizier, was used to facilitate the LOO analysis. Table 5 provides measures of error for each cross-validation run on the alpha–beta regression equations. A Mean Squared Error (MSE) was computed by determining the average of all squared differences between the original regression model and each of the LOO observations. A weighted average of absolute error is provided as the mean absolute error. The RMS (root mean square) error estimate displays a measure of the magnitude of variation. The three sets of error estimations indicate that in all instances the regression model with the least amount of error is the full three predictor model (Eq. (1)).

With the process of validation completed using the LOO cross-validation method, two examples show the application of the equations to an avalanche path in the highway corridor and an avalanche path in the backcountry.

4.3. Example of applying the three predictor Eq. (2) to a highway avalanche path

The next step was to apply the regression model to a selected avalanche path in the highway corridor (referred to as avalanche path B) that had not been used to generate the equations to compare the model's prediction with an observed maximum runout. This path was selected based on a well-known maximum runout. Avalanche path B is located on the west side of Rogers Pass and not one of the paths used to develop the regression equations.

In order to determine the location of the predicted runout ($\hat{\alpha}$) on an avalanche profile, the intersection must be determined between the line starting at the top of the avalanche profile and traveling downwards at the predicted alpha angle ($y=mx+b$) to the point on the DEM profile. Determining this runout point represents a special challenge as the angle is not a simple straight line shot down from the top of the mountain (such as could be solved in GIS with a simple line-of-sight function) but rather the avalanche profile curves and bends laterally down the slope. Determining the predicted runout was accomplished on profile graphs where the differing alpha predicted equation lines, as derived from the previous regression model equations, were plotted and intersected with the DEM profile. The intersection points of these lines were determined using the calculus functions in Alentum's Advanced Grapher software (version 2.11) and then transferred to GIS to be mapped.

Using the low resolution data (as no high resolution data was captured for this area) the α observed for avalanche path B, based on the historical database and verified by the avalanche expert, is 23.7°. In Fig. 4, avalanche path B is shown with the starting zone highlighted at the top of the path, the avalanche path outline and the centreline of the avalanche profile. The observed maximum runout has a horizontal distance of 2936 m and elevation of 939 m. As presented in Table 6, the $\hat{\alpha}$ predicted (23.4°), using the 3 predictor Eq. (2), intersected with the DEM profile resulting in a horizontal distance of 2979 m and elevation of 936 m. The difference between this alpha predicted and the observed alpha is 0.3° and within the standard deviation of 0.94°. This leaves an overestimate of a 43 m horizontal distance and a 3 m elevation drop as measured on the DEM profile. At the bottom of the image in Fig. 4, in the runout zone, the profile centreline slightly extends the 43 m past the expert identified runout maximum. It is important to recognize that this particular path has an avalanche defence mechanism of mounds that can shorten the runout.

4.4. Example applying the beta only Eq. (3) in the backcountry

The alpha–beta regression equation was also applied to an avalanche path in the backcountry. In Fig. 5, the avalanche path is delineated including a profile centreline. The beta point is shown along the avalanche centreline and the calculated runout extent is marked by an *X* at the furthest reach of the avalanche centreline. The avalanche path outline is based on vegetation to estimate the

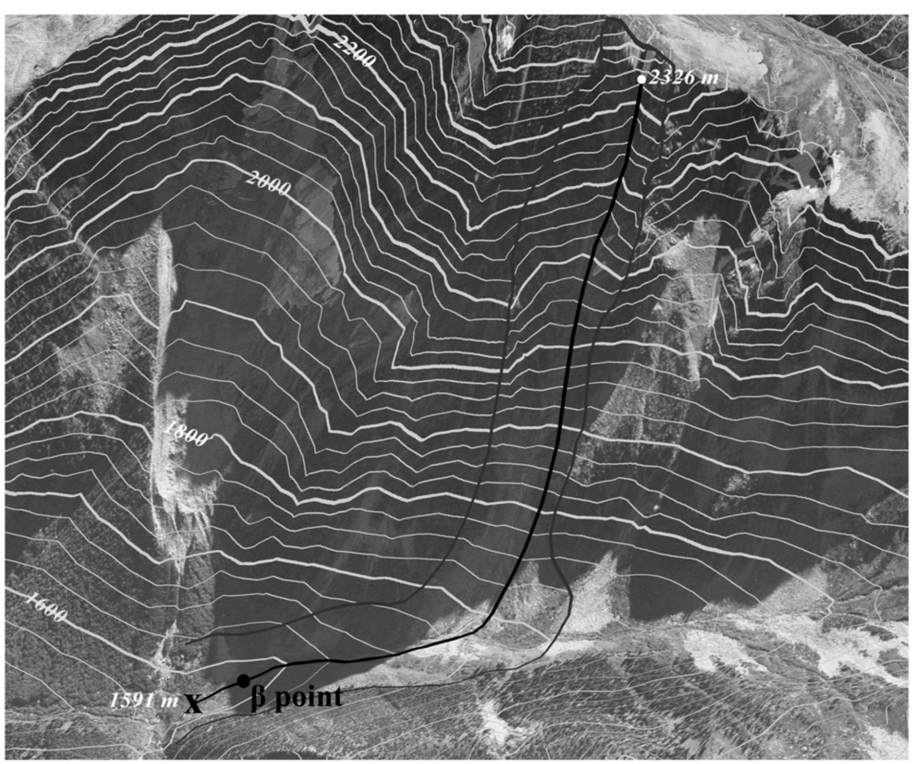

Fig. 5. Avalanche path in the Connaught Creek backcountry skiing area. Beta point is identified by the dot along the centreline profile where the slope declines to 10°. The alpha–beta regression model runout extent is represented by the furthest reach of the avalanche centreline and marked with a X. Image is draped on the high resolution DEM.

Table 7
Comparison of alpha–beta runout models

Country	Relation	R^2	S	N	Mean α	Mean H (m)	Mean β
Norway	$\hat{\alpha}=0.93\beta$	0.86	2.1°	192	n/a	n/a	n/a
Austria	$\hat{\alpha}=0.946\beta-0.83°$	0.92	1.5°	80	n/a	n/a	n/a
Iceland	$\hat{\alpha}=0.85\beta$	0.52	2.2°	44	n/a	n/a	n/a
Canada: Columbia Mountains	$\boldsymbol{\hat{\alpha}=0.93\beta}$	**0.89**	**1.1°**	**35**	**28.5°**	**946**	**30.6°**
Rockies/Purcells	$\hat{\alpha}=0.93\beta$	0.75	1.75°	126	27.8°	869	29.8°
Coast Mountains	$\hat{\alpha}=0.90\beta$	0.74	1.70°	31	26.8°	903	29.5°
Catalan Pyrenees	$\hat{\alpha}=0.86\beta+1.05°$	0.75	1.98°	64	24.7°	753	27.3°
Alaska	$\hat{\alpha}=0.86\beta$	0.58	n/a	52	25.4°	765	29.6°
Colorado	$\hat{\alpha}=0.80\beta$	0.50	n/a	130	22.6°	641	27.4°
Sierra Nevada	$\hat{\alpha}=0.76\beta$	0.60	n/a	90	20.7°	590	26.1°
Western Norway	$\hat{\alpha}=0.90\beta$	0.87	n/a	127	29.4°	827	32.6°

Model from this research is highlighted in bold. Data Sources: (Furdada and Vilaplana, 1998; Johannesson, 1998; Lied et al., 1995; McClung et al., 1989; Mears, 1988; Nixon and McClung, 1993).

avalanche path lateral extents. For ease of calculation in the backcountry the simplified Eq. (3) using beta as the only predictor was used to calculate the runout distance.

5. Discussion

The primary objective of this research was to produce a runout model to approximate a 10 year return period. For a 40 year observation period (L), the encounter probability (E) that an avalanche has been observed with a return period (T) of 10, 50, 100 years is 0.98, 0.55, and 0.33, respectively. The relationship between E, T and L is expressed as $E=1-(1-1/T)^L$ (LaChapelle, 1966; McClung, 1999). Hence the return period for the modelled avalanches can be labelled 10+ years. This provides a reasonable estimate of extreme avalanche events and hence the runout model produced is suitable for mapping backcountry environments. It is possible that the 40 year observation period may include shorter scale climatic fluctuations than a 100 year observation period typically used for residential planning. We have not analyzed such climatic fluctuations but assume that the effect on runout is weak.

The model should also be useful for avalanche paths controlled by explosives in the Columbia Mountains. New roads for mining or forestry in the region may be able to utilize the equations developed to predict runout maximums relevant to their operations.

The two compared DEM resolutions did not significantly affect the alpha–beta regression equations. This could be a combined result of using the same control data for the two datasets and is likely further impacted by the fact that beta is the strongest predictor in the regression models and represents an average angle (determined from top and bottom coordinate locations) that is not strongly influenced by variations in DEM resolution. Further, H and $Hy''\theta$ based on 4th or 2nd degree polynomial functions derived from the points extracted along the avalanche profiles provides a generalization of the curvature of the avalanche profile which may not reflect small variations in terrain between the DEM datasets.

In estimating the position of runout values using a predicted alpha angle, the DEM profile line will yield the most accurate result. As discussed in Appendix A, the 2nd degree polynomial does not accurately represent the runout portion of the avalanche profiles, so an alpha value positioned on the 2nd degree polynomial can result in a placement that is far removed from the actual observed maximum runout extent. If necessary the 4th degree polynomial provides a better estimate of the avalanche profile line and could potentially be extrapolated to estimate maximum runout position in cases of field collected data points where coordinates were unavailable beyond a certain maximum.

The equations developed for this study provide interesting comparison with equations from other countries (Table 7). Similar to other alpha–beta runout models applied worldwide, for this dataset alpha is correlated more strongly to beta than to other predictor variables. McClung et al. (1989) and McClung and Mears (1991) indicate that extreme runout from mountain ranges vary depending upon the terrain properties of the different ranges and are irrespective of snow climate. The alpha–beta runout models from the Columbia Mountains (Eq. (3)), the Rockies/Purcells and the runout model from Norway (with extreme events and short avalanche paths removed) are identical, although the applicable return periods differ. The paths from Canada and Norway are distinguished by steeper slopes, a greater vertical drop and shorter runout than the other mountain ranges in Table 7. This corresponds to findings from McClung et al. (1989) in which a scale effect was noticed whereby greater vertical drop was associated with shorter runout-ratio. To address the scale effects a partitioning of data into elevation categories to determine distinct runout models may be one possible solution as suggested by Nixon and McClung (1993). The Coast Mountains in Canada, the Catalyn Pyrennes, and the Western Norway dataset stand out as possible exceptions to the scale effect; where large mean vertical drop results in longer runout than other mountain ranges with similar mean vertical displacement. However, due to the scale effects within the individual datasets, differences in how the data were collected and applicable return period it is very difficult to compare the runout models from the different mountain ranges. For the runout model produced with the dataset in this study it is recommended to only apply this model in mountain ranges with similar terrain and a similar range of vertical displacement values. The model derived from this research is aimed at mapping backcountry environments and is not intended to estimate 100–300 year runouts required for human infrastructure planning and management.

In Canada, the traditional method for determining alpha–beta regression models is to conduct field surveys. GIS offers another strategy for collecting data on those avalanche slopes that may be difficult to survey in the field. In Norway the runout mapping is prepared in the lab first using GIS and supplemented by subsequent ground-truthing in the field combining the strengths of both computer and field methods (Lied et al., 1989). Further work would be to completely automate the process in GIS as well as compare runout results from the alpha–beta model with dynamic models.

A new form of avalanche terrain evaluation is being explored explicitly for Canadian backcountry users known as the Avalanche Terrain Exposure Scale (ATES) (Statham et al., 2006) and is being packaged for use in conjunction with the Canadian avalanche bulletin (Haegeli and McCammon, 2006). One of the ATES parameters is whether a backcountry user is travelling in an avalanche runout zone. The model developed from this study is being assessed for runout estimation for ATES terrain mapping.

5.1. Potential sources of error

There are several potential sources of error including: characteristics of the avalanche profiles, avalanche control in the model-building dataset, the historical database and limitations of photogrammetry. The highway corridor dataset that is used to generate the regression models is controlled with explosives and, as a result, may underestimate runout extent when the derived equation is applied to the backcountry. The control activity is likely to encourage more frequent avalanching and thus larger avalanche events with an increased runout are less likely. As well, the historical database captures only 40 years of data, thus avalanche events with longer return periods may not have been recorded.

Certain characteristics of the avalanche profiles need to be recognized such as the presence of barriers like the rail and road line and, to a limited degree, avalanche defence structures. These

Table A.1
Comparison of H values from the 2nd and 4th degree polynomials to the actual measured drop

High resolution dataset (n=35)	Average	For Path A	For Path B
Vertical drop (DEM)	946	1025	1287
H (4th order) (m)	950	1061	1289
H (2nd order) (m)	1187	1571	1438

barriers may reduce the predicted alpha values and hence runouts. Other characteristics include adjacent paths with overlapping runouts and hence the same maximum observed runout. In addition, the avalanche paths along the highway corridor runout onto broader and flatter valley bottom than the backcountry valleys. This has the potential to reduce the underestimation of backcountry runout maximums.

Two main issues in regards to photogrammetry interpretation and processing are the difficulty in identifying features in areas of shadow in air photos and in regards to the same control data being used to georeference air photos for both the low resolution dataset and the high resolution dataset. Mountains cast shadows, particularly on north-facing slopes. Although every attempt was made to capture DEM data from a variety of aspects, the shadowing effect made data capture impossible on some north-facing slopes and thus may bias the data used to generate the regression model.

Differences in elevation values are reduced between the high and low resolution datasets since the same control was used to reference the air photos. This British Columbia control data was the best available georeferencing. Yet the point of creating the high resolution DEM was to increase the level of terrain detail from the low resolution dataset. Other resolution issues include the maximum runout extents as recorded in the historical database which was often measured in extent beyond the highway and were not measured to within 5 m accuracy.

6. Summary and conclusions

Alpha–beta statistical snow avalanche runout models were developed for Glacier National Park, Canada. Models with three predictors (β, H and $Hy''\theta$) and a simple single predictor model (β only) were derived and cross validated. Similar to other studies referenced, the simplified β only model derived for this research has a high R^2 value close to that of the three predictor equations. The models were derived from both high and low resolution DEM datasets. A comparison of the models derived from the two datasets revealed no significant difference. Thus, DEM datasets at 25–30 m horizontal resolution would appear to be sufficient for determining alpha–beta runout calculations.

A third model was explored by modifying the alpha–beta model with a β angle referenced at 24° instead of the commonly used 10° β point. In Jones and Jamieson (2004), the β reference point, that is normally defined as the point at which the slope angle of the avalanche path decreases to 10°, was modified to 24° to accommodate short slopes of less than 600 m. For avalanche mapping, recreationists are often concerned with the runout from shorter slopes. In the backcountry where the regression model is applied, the valley bottom is less broad than the valley through which both the Canadian Pacific Railway (CPR) and TCH run. In these circumstances, it was found that 10° slopes occurred less often in the valley bottom or runout zone. The model with the β point at 24° did not have as high R^2 values as the other two predictor models.

Two examples demonstrate the model's applicability; one of a highway path outside the dataset used to derive the model and the second on an avalanche path in the backcountry. Although in contrast to the highway corridor, backcountry areas are not exposed to explosive-controlled avalanches, we assume that the regression equation will provide a reasonable estimate of runouts with a return period of approximately 10+ years.

Acknowledgements

The authors would like to acknowledge and thank Bruce McMahon, Senior Avalanche Officer, Parks Canada, for the many hours he contributed to identifying topographic features in the GIS for this study. We would also like to thank Dr. Tak Fung of the University of Calgary for the statistical analysis support, Janet Rose for her assistance with stereo photogrammetry and DEM generation and Laura Bakermans for sharing her findings on short avalanche paths. The Selkirk College Geospatial Research Centre is acknowledged for providing leading-edge infrastructure for the research. Support for this research was provided by an anonymous donor grant through the University of Calgary and is gratefully acknowledged. Finally, the authors would like to thank Editor G. Timco and two anonymous reviewers for their helpful comments that contributed to the improvement of the manuscript.

Appendix A. Comparing the effect of 2nd and 4th order polynomial fits on runout

In examining the best line of fit for the avalanche profiles of this dataset, the use of both 2nd and 4th order polynomials was explored.

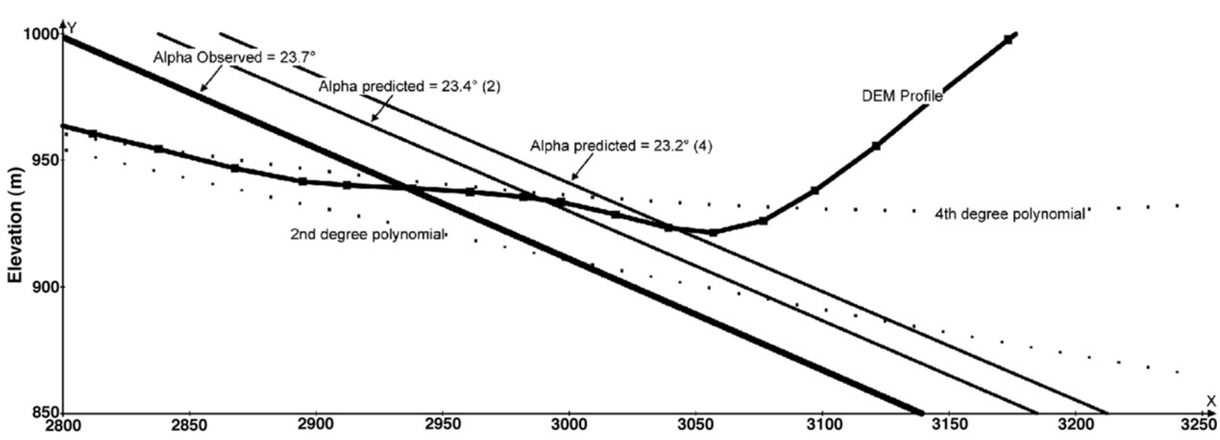

Fig. A.1. Avalanche path B runout zone comparing predicted alpha values as measured on the 2nd degree polynomial, 4th degree polynomial and the DEM profile. Table A.2. provides a measure of the distances from the observed alpha location.

Table A.2
Avalanche path B with calculated differences between the predicted alpha positions as measured on the 2nd degree polynomial, the 4th degree polynomial and the DEM profile

Equation and predicted alpha		2nd degree polynomial		4th degree polynomial		DEM profile	
Eq.	$\hat{\alpha}$ (°)	Distance (m)	Elevation (m)	Distance (m)	Elevation (m)	Distance (m)	Elevation (m)
(2)	23.4	146	−45	46	−1.4	43	−3.3
(4)	23.2	204	−55	85	−4.4	113	−17
(7)	22.7	328	−77	164	−8.2	205	−25

The purpose was to find a function that accurately represented the DEM profile in order to determine the regression parameters of H and $Hy''\theta$ necessary for the alpha–beta runout model. Second, a comparison was made between the position of the predicted alpha or maximum runout position by using extended 2nd and 4th order functions and the DEM profile.

To create 2nd and 4th order polynomials from the avalanche profile, the coordinate locations from the top of the profile to the expert identified maximum runout were utilized. Table A.1 compares the H values from the polynomials to the actual measured drop on the DEM profile. H as measured on the 4th order polynomial is much closer to the DEM measured vertical drop on average for the 35 avalanche profiles and for paths A and B. Fig. A.1 represents the runout portion of avalanche path B and shows the 2nd degree function dipping below the DEM profile. However this appears to be similar for other datasets and a difference in the H value measured on the 2nd degree polynomial compared to the true vertical drop is expected (Bakkehoi et al., 1983). Further, for this dataset, H and $Hy''\theta$ as measured on the 2nd degree polynomial are significant contributors along with beta to predict alpha (Section 4.1).

An important distinction must be made between calculating regression parameters from the 2nd degree polynomial for the regression equation and actually finding a final alpha predicted position on the avalanche profile. Ideally the extended DEM profile would be used to calculate the position at which the predicted alpha intersects it. To demonstrate the importance of using the DEM profile, Fig. A.1 displays the 2nd and 4th degree polynomial fits for avalanche path B in the runout zone. The 4th degree provides a much closer representation to the DEM avalanche profile and could potentially be used to estimate the position of alpha. The differences in the runout zone between the polynomial functions and the DEM profile is detailed in Table A.2. For example, the full runout model for avalanche path B as based on the 4th degree model is 46 m too long and 1.4 m too low; in contrast, the 2nd degree function for the same value yields an overestimate of 146 m too long and 45 m too low. Note that the dip in the DEM profile line (Fig. A.1) is due to the presence of a stream bed.

Thus, for this dataset, measuring H and $Hy''\theta$ on a 2nd order function contributed to finding an alpha–beta runout model which, along with beta, was used to estimate maximum runout. Finding the position of this point is ideally found along an extended DEM profile or if points beyond the surveyed maximum runout are not available, the 4th degree polynomial would be better to use than a 2nd degree polynomial as it provides a better fit for the runout portion of the profile.

References

Bakkehoi, S., Domaas, U., Lied, K., 1983. Calculation of snow avalanche runout distance. Annals of Glaciology 4, 24–29.

Bovis, M.J., Mears, A.I., 1976. Statistical prediction of snow avalanche runout from terrain variables in Colorado. Arctic and Alpine Research 8 (1), 115–120.

Canadian Avalanche Association, 2002. Guidelines for Snow Avalanche Risk Determination and Mapping in Canada. Canadian Avalanche Association, Revelstoke, BC. 24 pp.

Delparte, D., 2008. Avalanche Terrain Modeling in Glacier National Park, Canada. PhD Thesis, University of Calgary, Calgary, AB, Canada, 179 pp.

Fujisawa, K., Tsunaki, R., Kamiishi, I., 1993. Estimating snow avalanche runout distances from topographic data. Annals of Glaciology 18, 239–244.

Furdada, G., Vilaplana, J.M., 1998. Statistical predication of maximum avalanche run-out distances from topographic data in the western Catalan Pyrenees (northeast Spain). Annals of Glaciology 26, 285–288.

Haegeli, P., McCammon, I., 2006. Avaluator Avalanche Accident Prevention Card. Canadian Avalanche Association (CAA), Revelstoke, BC, Canada.

Harbitz, C., Harbitz, A., Nadim, F., 2001. On probability analysis in snow avalanche hazard zoning. Annals of Glaciology 32, 290–305.

Johannesson, T., 1998. Icelandic avalanche runout models compared with topographic models used in other countries. In: Hestnes, E. (Ed.), Proceedings of the Anniversary Conference 25 Years of Snow Avalanche Research, Voss, 12–16 May 1998. Norwegian Geotechnical Institute, Oslo, pp. 43–52.

Jones, A.S.T., Jamieson, B., 2004. Statistical avalanche runout estimation for short slopes. Annals of Glaciology 38, 363–372.

LaChapelle, E.R., 1966. Encounter probabilities for avalanche damage, USDA Forest Service, Miscellaneous Report 10.

Lied, K., 1998. Snow avalanche experience through 25 years at NGI. In: Hestnes, E. (Ed.), Proceedings of the Anniversary Conference 25 Years of Snow Avalanche Research, Voss, 12–16 May 1998. Norwegian Geotechnical Institute, Oslo, pp. 7–14.

Lied, K., Bakkehoi, S., 1980. Empirical calculation of snow-avalanche run-out distance based on topographic parameters. Journal of Glaciology 26 (94), 165–177.

Lied, K., Sandersen, F., Toppe, R., 1989. Snow avalanche maps for the Norwegian army. Annals of Glaciology 13, 170–174.

Lied, K., Toppe, R., 1989. Calculation of maximum snow-avalanche runout distance by use of digital terrain models. Annals of Glaciology 13, 164–169.

Lied, K., Weiler, S., Bakkehoi, S., Hopf, J., 1995. Calculation methods for avalanche run-out distance for the Austrian Alps. The contribution of scientific research to safety with snow, ice and avalanche. ANENA, Grenoble, France, pp. 63–68.

McClung, D., 1999. The encounter probability for mountains slope hazards. Canadian Geotechnical Journal 36 (6), 1195–1196.

McClung, D., 2000. Extreme avalanche runout in space and time. Canadian Geotechnical Journal 37, 161–170.

McClung, D., Lied, K., 1987. Statistical and geometric definition of snow avalanche runout. Cold Regions Science and Technology 13, 107–119.

McClung, D., Mears, A.I., 1991. Extreme value prediction of snow avalanche runout. Cold Regions Science and Technology 19, 163–175.

McClung, D., Mears, A.I., 1995. Dry flowing avalanche run-up and run-out. Journal of Glaciology 41 (138), 359–372.

McClung, D., Mears, A.I., Schaerer, P., 1989. Extreme avalanche runout: data from four mountain ranges. Annals of Glaciology 13, 180–184.

Mears, A.I., 1988. Comparisons of Colorado, Eastern Sierra, Coastal Alaska, and Western Norway Runout Data. International Snow Science Workshop (ISSW), Wistler, BC. pp. 232–238.

Mears, A.I., 1992. Snow Avalanche Hazard Analysis for Land-Use Planning and Engineering, Bulletin 49. Colorado Geological Survey. Geological Survey, Dept. of Natural Resources, Denver, Colorado. 54 pp.

Nixon, D.J., McClung, D., 1993. Snow avalanche runout from two Canadian mountain ranges. Annals of Glaciology 18, 1–6.

Paternoster, R., Brame, R., Mazerolle, P., Piquero, A., 1998. Using the correct statistical test for the equality of regression coefficients. Criminology 36 (4), 859–866.

Schleiss, V.G., 1989. Rogers Pass Snow Avalanche Atlas, Glacier National Park, British Columbia, Canada. Canadian Parks Service, Revelstoke, B.C. 313 pp.

Statham, G., McMahon, B., Tomm, I., 2006. The Avalanche Terrain Exposure Scale. In: Gleason, J.A. (Ed.), International Snow Science Workshop (ISSW), Telluride, CO, pp. 491–497.

Toppe, R., 1987. Terrain models: a tool for natural hazard mapping. In: Salm, B., Gubler, H. (Eds.), Avalanche Formation, Movements and Effects (Proceedings of the Davos Symposium, September 1986). International Association of Hydrological Sciences (IAHS), Wallingford, UK, pp. 629–638.

Weir, P., 2002. Snow Avalanche Management in Forested Terrain, Land Management Handbook No. 55. Res., Br., B.C. Min. For., Victoria, BC. 208 pp.

Wilks, D.S., 1995. Statistical Methods in the Atmospheric Sciences: An Introduction 59. Academic Press, San Diego, CA. 465 pp.

Woods, J.G., 1983. Snow War. An illustrated history of Rogers Pass, Glacier National Park, British Columbia. National and Provincial Parks Association of Canada. 52 pp.